

Roton Parametric Resonance

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Abstract

Parametric excitation of rotons by oscillating electric field exhibits a narrow resonance at the roton minimum frequency. The resonance region width is in good agreement with experimental results on the microwave absorption in superfluid helium.

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1 Introduction

Recent experiments with a whispering gallery resonator submerged in superfluid ^4He produce striking result. The electromagnetic damping exhibits an ultra-narrow resonance [1, 2, 3]. The resonance frequency depends on the temperature and tracks precisely the roton energy gap.

Such results were truly unexpected:

- (A) Energy spectrum of helium is continuous, excitations exist both below (phonons) and above (phonons and rotons) the roton energy gap. The ultra-narrow absorption line can not be explained with plain single-particle density of states peculiarities. To the best of our knowledge, no realistic explanation has been suggested for this.
- (B) Helium is believed to be perfectly neutral and has relatively low susceptibility ($\epsilon = 1.05$). It is therefore not obvious how electromagnetic oscillations interact with superfluid excitations. Conceivable scenarios include polarization of liquid by motion [4, 5, 6] or vulgar acoustic emission by mechanic vibrations of the resonator walls due to electrostriction.
- (C) Creation of a roton requires significant momentum. Photon momentum is much smaller and can not account for the momentum conservation. This process was therefore thought to be possible [1] only in the immediate vicinity of the wall which breaks translation invariance.

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These difficulties motivated present research.

To deal with the problem (A) we assume that observed effect is essentially collective. Particularly, rotons are bosons, emission of new ones is stimulated by those already present. It is important, that new rotons are emitted into the same quantum state. The avalanche occurs if the gain (number of rotons emitted per unit time) is bigger than the losses (number of rotons leaving active volume per unit time). The latter is proportional to the roton velocity and is arbitrary small near the minimum energy.

Similar phenomenon is realized in lasers, where photons are usually kept in a resonant cavity to stimulate emission of subsequent photons. One does not need a resonator for rotons, they remain in the active volume due to their slowness.

Without excessive speculations we stick to the traditional [7] expression $\epsilon \mathbf{E}^2 / (8\pi)$ for the energy density in the electric field. This term amounts to the field-dependent change in the energy of elementary excitation. Particularly for the rotons [8]

$$\epsilon = \Delta + \frac{(p - p_0)^2}{2m} + \alpha_{ij} \frac{E^i E^j}{2}, \quad (1)$$

where $\Delta = 8.7 \text{ K}$, $p_0 = 2 \cdot 10^{-19} \text{ g} \cdot \text{cm/s}$, and $m = 0.16 m_{\text{He}}$ are the roton spectrum parameters. Precise value of the factor α (roton polarizability) is unknown¹ but can be estimated [8] as following

$$\alpha \sim \frac{(\epsilon - 1)^2 p_0^2}{4\pi\rho\Delta} \sim 4.5 \cdot 10^{-26} \text{ cm}^3, \quad (2)$$

where $\rho = 0.145 \text{ g/cm}^3$ is helium density.

Suppose the electric field oscillates with the frequency $2\pi f$. The roton energy (1) “oscillates” twice as fast because the last term is quadratic with respect to the electric field. This means that the electric field parametrically pumps the rotons and their population grows exponentially if the pumping frequency $4\pi f$ is close to the double roton frequency $2\omega = 2\epsilon/\hbar$. Note also, that the problem of seemingly unconserved momentum (C) is irrelevant for the parametric roton excitation: two rotons are created simultaneously.

2 Resonance width

As explained above, a coherent roton state with large occupation number is formed in a parametric resonance. This enables a classical [10] treatment of the roton field. In a mechanical system with the time-dependent intrinsic frequency $\omega(t) = \omega + \gamma \cos 4\pi f t$, the parametric resonance occurs if

$$|4\pi f - 2\omega| < \sqrt{\gamma^2 - 4\lambda^2}. \quad (3)$$

¹Experimentally, the resonance line is split in two when a static electric field is applied. We may attribute the second order term in the frequency displacement of a resonance component to the polarization of rotons. This gives [9] $\alpha = 2.1 \cdot 10^{-25} \text{ cm}^3$.

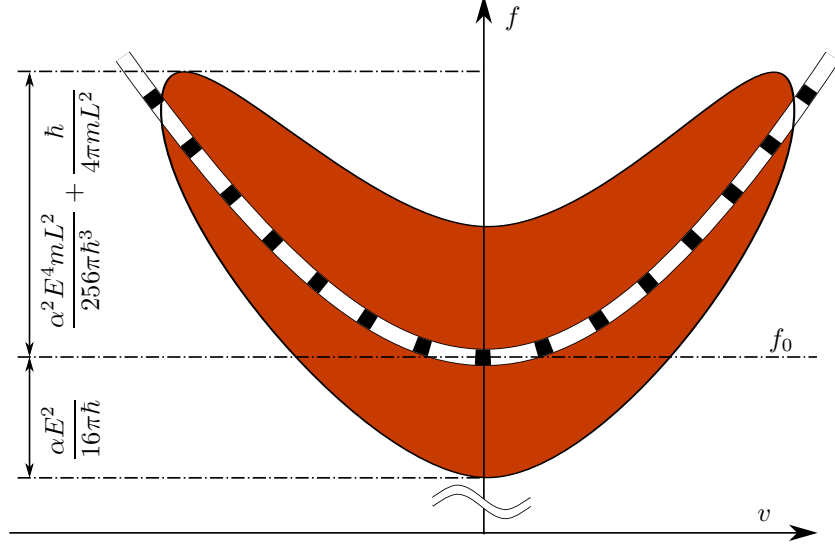


Figure 1: Grin-like area corresponds to the parametric instability (6) region (not to scale). Roton dispersion $2\pi\hbar f = mv^2/2$ is plotted with dashed line.

The damping decrement λ for rotons is determined by the time they spend in the active volume. If the pumping field is confined in a volume with linear dimension L , then

$$\lambda \sim v/L, \quad (4)$$

where $v = (p - p_0)/m$ is the roton velocity. Assuming $E(t) = E \cos 2\pi f t$ we get

$$\gamma \sim \frac{\alpha E^2}{4\hbar}. \quad (5)$$

Substituting (1), (4), and (5) into (3) we obtain

$$\left| 4\pi f - \frac{2\Delta}{\hbar} - \frac{mv^2}{\hbar} \right| \lesssim \sqrt{\left(\frac{\alpha E^2}{4\hbar} \right)^2 - \frac{4v^2}{L^2}}. \quad (6)$$

This inequality can be satisfied (at varying velocities, see Fig.1) if

$$-\frac{\alpha E^2}{16\pi\hbar} \lesssim f - f_0 \lesssim \frac{\alpha^2 E^4 m L^2}{256\pi\hbar^3} + \frac{\hbar}{4\pi m L^2}, \quad (7)$$

where $f_0 = \Delta/(2\pi\hbar) \sim 180$ GHz.

3 Discussion

In the whispering gallery resonator experiments the absorption line width at low temperature could not be reliably measured. Instrumental frequency resolution

was on the order of 50 kHz. At higher temperature the line width increases and reaches 350 kHz.

In these experiments the oscillating electric field reaches as much as $E \sim 200$ V/mm on the resonator wall and decays exponentially outward in helium. The length scale is on the order of magnitude of the microwave length $L \sim 1$ mm. Substituting these numbers in (7) we get

$$-40 \text{ Hz} \lesssim f - f_0 \lesssim 45 \text{ kHz}.$$

This gives the full resonance region width of about 45 kHz at low temperature. Exact spectral line shape (not Lorentzian) is determined by the nonlinear terms.

Several factors should contribute to the parametric instability widening at finite temperature. Roton scattering explicitly increases the roton decay and the natural roton linewidth (neglected above). It also reduces their active volume escape rate, effectively retarding them.

We emphasize, that a coherent roton state (a ROTON) is formed by the parametric excitation. In this sense the electromagnetic resonator and the superfluid around it work as a “laser of rotons” (raser).

This is the roton slowness that is responsible for the narrow resonance. Another stationary point of the superfluid dispersion curve corresponds to maxons, the same effect is expected at the respective frequency $f_M \sim 300$ GHz. Present analysis is not specific to superfluid, other quasiparticles (magnons in magnetics, photons in metamaterials, *etc.*) with zero velocity will exhibit similar behavior. Note, that possibility of parametric excitation of magnons [11] and photons [12] is well established.

Higher order parametric resonances are possible at fractional frequencies. It would be interesting to perform accurate measurement of the resonator quality factor near $f_0/2$, $f_0/3$, *etc.* Another contribution is the result of linear coupling to the electric field [13] due to the roton quadrupole moment. This will lead to the double frequency resonance at $2f_0$.

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